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A ROBUST ESTIMATOR FOR POLYNOMIAL PHASE SIGNALS IN NON GAUSSIAN NOISE USING PARALLEL UNSCENTED KALMAN FILTERS

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ABSTRACT

In this paper, we address the problem of the estimation of polynomial phase signals (PPS) in "ϵ-contaminated" impulsive noise using Kalman filtering technique. We consider an original estimation method based on the exact non linear state space representation of the signal by using the unscented Kalman filter (UKF) instead of the classical approach which consists in the linearization of the system of equations and then applying the extended kalman filter (EKF). The observation noise's probability density function is assumed to be a sum of two-component Gaussians weighted by the probability of appearance of the impulsive and gaussian noises in the observations. We propose to use two unscented Kalman filters operating in parallel (PUKF) as an alternative to the classical methods which generally handle the impulsive noise by using either clipping or freezing procedures. Simulation results show that the PUKF is less sensitive to impulsive noise and gives better estimation of signal parameters compared to the recently proposed algorithms.

1. INTRODUCTION

Polynomial phase signals arise in many natural phenomena like seismic and bat echolocation signals, but are mainly known in engineering applications such as in radar, sonar and time-varying channels in telecommunication. The estimation of polynomial phase signals is a well known problem in signal processing and has received considerable interest in literature. Most of the proposed methods for the estimation of the parameters of PPS assume that the signal is affected by additive Gaussian noise. These methods rely on maximum likelihood principle (ML), LMS/RLS estimators, and time-frequency representations [1]. A widely used approach to estimate the signal parameters is the Kalman filter [2] which is the optimal tracking algorithm when the signal models are assumed linear and both state and observation noises are additive Gaussian [3]. When these assumptions do not hold, that is when one has to tackle non linear models, then the Extended Kalman Filter (EKF) [4] is used by considering a local linearization using first order Taylor expansion of the non-linear equations. Many algorithms based on the EKF with various configurations have been proposed. For exemple, the authors in [5] proposed a state space model obtained by incorporating spatial information consisting of the corrupted signal and its delayed version using two sensors, then the PPS parameters were estimated using two EKFs in cascade. Still, in many situations the observation noise is non Gaussian. In [6], the authors considered the estimation of chirp signals in additive/multiplicative non-Gaussian noise using ML and LSE estimators. Recently, we proposed an algorithm based on parallel EKFs (PEKF) for the estimation of chirp signals corrupted by ϵ-contaminated noise [7]. In this paper, we consider the estimation of the parameters of PPS corrupted by an additive noise based on a state-space representation using two unscented Kalman filters (UKF) operating in parallel. Without loss of generality, we consider in our approach, the exact non linear state-space model for mono-component PPS derived in [3] as it can be easily extended two multi-component PPS, however, we assume the additive noise is impulsive with a non Gaussian distribution to obtain a non linear/ non Gaussian state space model. The non Gaussian nature of the noise is motivated by the fact that many natural phenomena or man-made applications [8] such as atmospheric disturbances affecting HF communication are characterized by spikes with large amplitudes. These spikes significantly degrades the performance of most frequency tracking based algorithms in which noise is assumed Gaussian. As stated, we consider a practical model for the pdf of the impulsive noise is the sum of two weighted Gaussian density functions. One way to use Kalman filtering in impulsive environment is by either clipping the observation signal or by changing or freezing the Kalman gain estimation with respect to a defined threshold [9]. In order to overcome the limitations due to these techniques, we propose to use parallel unscented Kalman filtering, hence avoiding both thresholding and linearisation. The key idea of our proposed algorithm consists in replacing the EKF filters in [7] by two parallel UKFs so that each UKF is tuned on one Gaussian component and their estimates are weighted to produce the final state estimate.

This paper is organized as follows. In sections 2 and 3, we briefly present the models of the PPS and non Gaussian noise. Then, we introduce in section 4 the exact non linear and non Gaussian state space modelisation of PPS corrupted by impulsive noise. In section 5, we describe in detail the PUKF algorithm for the estimation of the parameters of the PPS. Section 6 provides simulation results and comparison with respect to the parallel extended Kalman (PEKF). Finally, we give some concluding remarks and perspective work in section 7.

2. THE POLYNOMIAL PHASE SIGNAL MODEL

The general expression of a polynomial phase signal of order M is given by

$$z(k) = A(k)\exp\{j\phi(k)\} = A(k)\exp\left\{j\sum_{i=0}^{M} a_i k^i\right\} + v(k) \tag{1}$$

where A is the amplitude of the signal, the a_i's $(i = 0, \ldots, M)$ are the phase coefficients; assumed real and unknown. In the sequel, the additive noise $v(k)$ is assumed complex non Gaussian with known parameters.

In this paper, we will consider the following signal modelization

$$y(k) = \Re\{z(k)\} = A(k)\cos(\phi(k)) + n(k) \tag{2}$$

Where the additive noise $n(k) = \Re\{v(k)\}$ is real non Gaussian with known parameters as expressed in next section.
The instantaneous frequency (IF) is defined as

$$f_i(k) = \frac{1}{2\pi}\frac{d\phi(k)}{dk} = \frac{1}{2\pi}\sum_{i=1}^{M} i\ a_i\ k^{i-1}. \tag{3}$$

3. NON GAUSSIAN NOISE : MIDDLETON MODEL

There exists many physical processes generating interference containing noise components that are impulsive in nature (e.g., atmospheric noise in radio links; and radar reflections from ocean waves, and reflections from large, flat surfaces including buildings and vehicles). The amplitude distributions of such returns are not Gaussian; this produces large-amplitude outliers in the observations. Impulsive noise profoundly degrades the performance of standard algorithms based on second order statistics, thus producing poor results.
The probability density function (pdf) of the noise $n(k)$ is given by the sum of two weighted Gaussians with variances σ_1^2 and σ_2^2 with $(\sigma_2^2 >> \sigma_1^2)$. Typically the ratio (σ_2^2/σ_1^2) is between 100 to 500 [10].

$$p(n(k)) = (1-\epsilon)\mathcal{N}(0,\sigma_1^2) + \epsilon\mathcal{N}(0,\sigma_2^2) \tag{4}$$

where the parameter ϵ corresponds to the probability of appearance of the impulsive noise. Figure 1 shows the pdf of the noise with respect to ϵ.

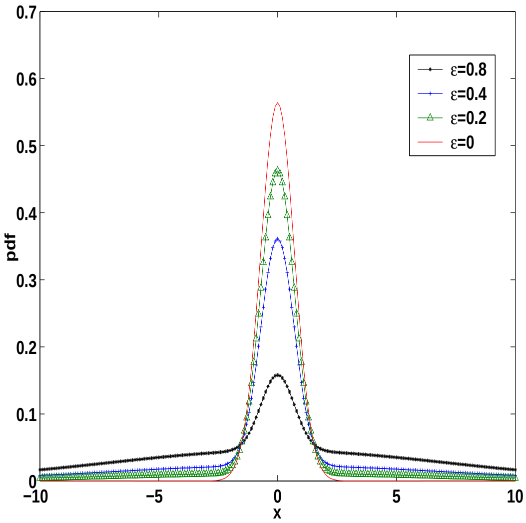

Fig. 1. The pdf of the noise for different values of ϵ

4. NON LINEAR STATE SPACE REPRESENTATIONS OF PPS

We consider the discrete signal $y(k)$ given by equation (2) to model by a polynomial phase signal of order M which is affected by an additive noise $n(k)$. In order to obtain the exact the state space representation, We define the following state vector

$$\mathbf{x}(k) = \begin{bmatrix} A(k) & \phi(k) & \Delta\phi(k) & \Delta^2\phi(k) & \dots & \Delta^M\phi(k) \end{bmatrix}^T. \tag{5}$$

where

$$\Delta\phi(k) = \frac{1}{2}(\phi(k+1) - \phi(k-1)) \tag{6}$$

and

$$\Delta^\ell\phi(k) = \frac{1}{2}(\Delta^{\ell-1}\phi(k+1) - \Delta^{\ell-1}\phi(k-1)) \tag{7}$$

for $\ell = 2...M$, and the observation equation is given by

$$y(k) = A(k)\cos(\phi(k)) + n(k) \tag{8}$$

which can be rewritten as

$$y(k) = x_1(k)\ \cos(x_2(k)) + n(k) \tag{9}$$

Now, assuming that the amplitude of the signal follows a random walk model driven by a Gaussian noise w with variance σ_w^2 as given below

$$A(k) = A(k-1) + w(k) \tag{10}$$

Hence, the state space model associated with the PPS can written as a linear state equation and a non-linear observation equation

$$\begin{array}{c} \mathbf{x}(k+1) = \mathbf{F}\mathbf{x}(k) + \mathbf{G}w(k) \\ y(k) = h(\mathbf{x}(k)) + n(k) \end{array} \tag{11}$$

where

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 1/1! & \dots & 1/M! \\ \vdots & \ddots & \ddots & \dots & \vdots \\ \vdots & \ddots & \ddots & 1 & 1/1! \\ 0 & \dots & \dots & 0 & 1 \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix} \tag{12}$$

The nonlinear function $h(\mathbf{x}(k))$ is

$$h(\mathbf{x}(k)) = x_1(k)\ \cos(x_2(k)) \tag{13}$$

5. ROBUST ESTIMATION BASED ON THE PARALLEL UKF ALGORITHM

Based on the formalism of [11], the author in [12] proposed a network of Kalman filters (NKF) in the case where the the observation noise is Gaussian while the state noise is non Gaussian. A modified version of this algorithm is proposed here, it is based on the computation of the a posteriori pdf of the state

$$p(\mathbf{x}(k)|Y^k) = \frac{p(\mathbf{x}(k)|Y^{k-1})p(y(k)|\mathbf{x}(k))}{\int p(\mathbf{x}(k)|Y^{k-1})p(y(k)|\mathbf{x}(k))d\mathbf{x}(k)} \tag{14}$$

where $Y^{k-1} = [y(k-1), y(k-2), \dots y(0)]$
The likelihood of the observation $p(y(k)|\mathbf{x}(k))$ is given by

$$\begin{aligned} p(y(k)|\mathbf{x}(k)) = (1-\epsilon)\mathcal{N}(y(k) - H(\mathbf{x}(k)), \sigma_1^2) + \\ \epsilon\mathcal{N}(y(k) - H(\mathbf{x}(k)), \sigma_2^2) \end{aligned} \tag{15}$$

The main idea is to approximate the densities $p(\mathbf{x}(k)|Y^k)$ and $p(\mathbf{x}(k)|Y^{k-1})$ by a weighted sums of Gaussian density functions

$$p(\mathbf{x}(k)|Y^k) = \sum_{i=1}^{\xi} \alpha_{i,k} \mathcal{N}(x(k) - \hat{x}_i(k), P_i(k)) \qquad (16)$$

and

$$p(\mathbf{x}(k)|Y^{k-1}) = \sum_{i=1}^{\xi'} \alpha'_{i,k} \mathcal{N}(x(k) - \hat{x}'_i(k), P'_i(k)) \qquad (17)$$

The predicted pdf $p(\mathbf{x}(k+1)|Y^k)$ for the next iteration is obtained using

$$p(\mathbf{x}(k+1)|Y^k) = \int p(\mathbf{x}(k+1)|\mathbf{x}(k)) p(\mathbf{x}(k)|Y^k) d\mathbf{x}(k) \quad (18)$$

Since the noise $w(k)$ is assumed Gaussian, we have

$$p(\mathbf{x}(k+1)|\mathbf{x}(k)) = \mathcal{N}(\mathbf{x}(k+1) - \mathbf{F}\mathbf{x}(k), GG^T\sigma_w^2) \qquad (19)$$

Following the mathematical development in ([12], see Appendix), we proposed the two parallel EKF algorithm for chirp signal estimation [7]. It is clear that in order to obtain a better estimate of the parameters, the EKF filters can be replaced by the UKF which do not need any linearisation of the state space equations of the model. The UKF uses the unscented transformation (UT) which allows the calculation of the statistics of a random variable undergoing a non linear transformation [13] [14].
Consider a random variable $\mathbf{x}$ of dimension L with mean $\overline{\mathbf{x}}$ and covariance $\overline{\mathbf{P}}_{\mathbf{x}}$, then $\mathbf{x}$ is propagated through a non linear function, $\mathbf{y} = f(\mathbf{x})$. In order to compute the statistics of $\mathbf{y}$, we form a set of sigma points according to :

$$\begin{aligned}
\mathcal{X}_0 &= \overline{\mathbf{x}} \\
\mathcal{X}_i &= \overline{\mathbf{x}} + \left(\sqrt{(L+\kappa)\,\overline{\mathbf{P}}_{\mathbf{x}}}\right)_i, \quad i = 1 \dots L \\
\mathcal{X}_i &= \overline{\mathbf{x}} - \left(\sqrt{(L+\kappa)\,\overline{\mathbf{P}}_{\mathbf{x}}}\right)_{i-L}, \quad i = L+1 \dots 2L
\end{aligned} \qquad (20)$$

The sigma points are then propagated through the function f, as $\mathcal{Y}_i = f(\mathcal{X}_i)$, which yields to the mean and covariance of $\mathbf{y}$

$$\overline{\mathbf{y}} = \sum_{i=0}^{2L} W_i \mathcal{Y}_i, \quad \mathbf{P}_{\mathbf{y}} = \sum_{i=0}^{2L} W_i \left(\mathcal{Y}_i - \overline{\mathbf{y}}\right)\left(\mathcal{Y}_i - \overline{\mathbf{y}}\right)^T \qquad (21)$$

The weights are computed by

$$W_0 = \frac{\kappa}{(\kappa + L)}, \qquad W_i = \frac{1}{2(\kappa + L)}, \quad i = 1 \dots 2L \qquad (22)$$

where κ is a scaling parameter and is set such that $\kappa + L = 3$ [13]. $\left(\sqrt{(L+\kappa)\,\overline{\mathbf{P}}_{\mathbf{x}}}\right)_i$ is the i^{th} column of the matrix square root. For more details on the theoretical aspects of the UT and the UKF, the reader can refer to [14], [13].

Hence, We obtain a system composed of two UKF filters where each filter is tuned on one Gaussian component of the noise characterized by (4).

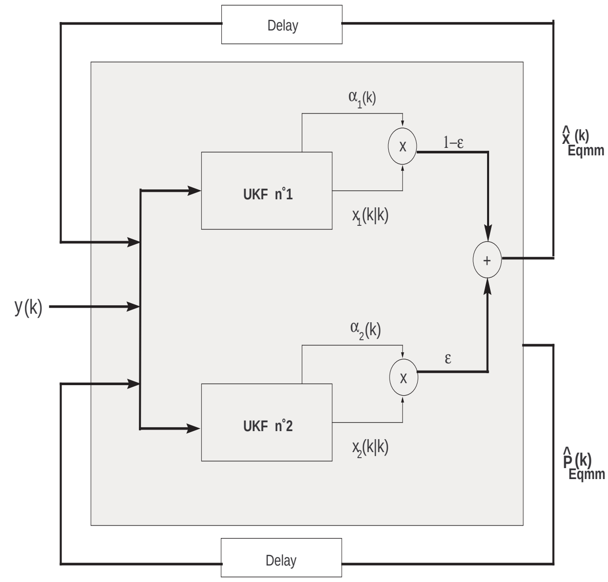

Fig. 2. Amplitude estimation

Table 1: Summary of the PUKF algorithm

Given the initial conditions
$\widehat{\mathbf{x}}_0 = E\left[\mathbf{x}_0\right]$

$$\overline{\mathbf{x}}_0 = \left[\begin{array}{ccc} \widehat{x}_0^T & \mathbf{0} & \mathbf{0} \end{array}\right]^T$$

$$\overline{\mathbf{P}}_0 = \left[\begin{array}{ccc} \mathbf{P}_0 & 0 & 0 \\ 0 & \mathbf{P}_w & 0 \\ 0 & 0 & \mathbf{P}_n \end{array}\right]$$

Where $\mathbf{P}_0$ is the initial state covariance matrix, and $\mathbf{P}_w$, $\mathbf{P}_n$ are the process noise and the measurement noise covariance matrices respectively. Then, one computes the sigma points as given in equation (20).

Compute for $k : 1, 2, 3 ...$

Prediction step
$\mathcal{X}_{i,j}(k|k-1) = \mathbf{F}\mathcal{X}_{i,j}(k-1|k-1) \quad i = 0, \dots, 2L$ and $j = 1, 2$
$\widehat{\mathbf{x}}_j(k|k-1) = \sum_{i=0}^{2L} W_i \mathcal{X}_{i,j}(k|k-1)$
$\widetilde{\mathbf{x}}(k|k-1) = \mathcal{X}_{i,j}(k|k-1) - \widehat{\mathbf{x}}_j(k|k-1)$
$\widehat{\mathbf{P}}_j(k|k-1) = \sum_{i=0}^{2L} W_i \widetilde{\mathbf{x}}(k|k-1)\widetilde{\mathbf{x}}(k|k-1)^T$
$\mathcal{Y}_{i,j}(k|k-1) = h(\mathcal{X}_{i,j}(k|k-1))$
$\widehat{y}_j(k|k-1) = \sum_{i=0}^{2L} W_i \mathcal{Y}_{i,j}(k|k-1)$

Filtering step
$\zeta_j^2(k|k-1) = \sum_{i=0}^{2L} W_i \left(\mathcal{Y}_{i,j}(k|k-1) - \widehat{y}_j(k|k-1)\right)^2 + \sigma_j^2$
$\mathbf{P}_{xy}(k|k-1) = \sum_{i=0}^{2L} W_i \widetilde{\mathbf{x}}(k|k-1)\widetilde{\mathbf{y}}(k|k-1)$
$\widetilde{\mathbf{x}}(k|k-1) = \mathcal{X}_{i,j}(k|k-1) - \widehat{\mathbf{x}}_j(k|k-1)$
$\widetilde{\mathbf{y}}(k|k-1) = \mathcal{Y}_{i,j}(k|k-1) - \widehat{y}_j(k|k-1)$
$e_j(k|k-1) = y(k) - \widehat{y}_j(k|k-1)$

$\widehat{\mathbf{x}}_j(k|k) = \widehat{\mathbf{x}}_j(k|k-1) + \mathbf{K}_j(k)e_j(k|k-1)$
$\widehat{\mathbf{P}}_j(k|k) = \widehat{\mathbf{P}}_j(k|k-1) - \mathbf{K}_j(k)\zeta_j^2(k|k-1)\mathbf{K}_j^T(k)$
$\mathbf{K}_j(k) = \mathbf{P}_{xy}(k)/\zeta_j^2(k|k-1)$

MMSE state estimation
$\beta_j(k) = \mathcal{N}(e_j(k|k-1), \zeta_j^2(k|k-1))$

$$\alpha_j(k) = \frac{\lambda_j \beta_j(k)}{\sum_{j=1}^{2} \lambda_j \beta_j(k)} \qquad \text{with} \quad \lambda_1 = 1-\epsilon, \quad \lambda_2 = \epsilon$$

$\widehat{\mathbf{x}}_{MMSE}(k) = \sum_{j=1}^{2} \alpha_j(k)\widehat{\mathbf{x}}_j(k|k)$
$\widehat{\mathbf{P}}_{MMSE}(k) = \sum_{j=1}^{2} \alpha_j(k)\widehat{\mathbf{P}}_j(k|k) + (\widehat{\mathbf{x}}_j(k|k) - \widehat{\mathbf{x}}_{MMSE}(k))(\widehat{\mathbf{x}}_j(k|k) - \widehat{\mathbf{x}}_{MMSE}(k))^T$

Finally, we estimate the parameters of the signal given by the vector $\theta = \left[\begin{array}{ccccc} A(k) & a_1 & a_2 & \ldots & a_M \end{array}\right]^T$ from the state estimates using the following relation [5]

$$\hat{\theta}(k) = \mathbf{A}\mathbf{F}^{-k}\widehat{\mathbf{x}}_{MMSE}(k) \tag{23}$$

where the matrix A is a diagonal :

$$\mathbf{A} = diag\left[\begin{array}{ccccc} 1 & 1 & 1/1! & \ldots & 1/M! \end{array}\right] \tag{24}$$

6. SIMULATION RESULTS

In this section, we give some simulation results for the estimation of PPS in non Gaussian noise based on the PUKF. We consider a signal PPS of order 2 of 1000 samples and sampled at a period equals 1. The true values of the signal parameters are as follows : $a_2 = 1.25 \times 10^{-3}$, $a_1 = 0.1$, and $a_0 = \frac{\pi}{2}$. The state noise $w(k)$ is zero mean Gaussian white noise with $\sigma_w^2 = 10^{-2}$. The non Gaussian noise $n(k)$ with pdf given by(4), has variance $\sigma_1^2 = 0.25$, and the ratio $\sigma_2^2/\sigma_1^2 = 500$. The initialization of the UKF is done in the same way as the EKF in [7]. In order to assess the gain in performance, we compare the MSE of the PUKF with the PEKF in [7]. We observe in figures 3 to 6 that the PUKF is more robust and gives better estimates of the chirp parameters for whole range of the SNR which is given between -5dB and 10dB. The SNR in our case is defined as

$$SNR = 10\log_{10}\left(\frac{A^2}{(1-\epsilon)\sigma_1^2 + \epsilon\sigma_2^2}\right) \tag{25}$$

7. CONCLUSION

We have presented an original approach based on the unscented Kalman filter for estimating the parameters of polynomial phase signals described by non linear state space model where the observation noise is non Gaussian. The estimation procedure is carried out by using two parallel UKF filters without resorting to threshold computation or linearisation as compared to other algorithms in literature. Through simulation results we showed the PUKF yields a significant improvement of the performance of both estimation and robustness. On the other hand, this algorithm allows the extension to multicomponent PPS signals and variable amplitude high order PPS. Furthermore, work on computing the Cramer-Rao bound for this case is currently under consideration.

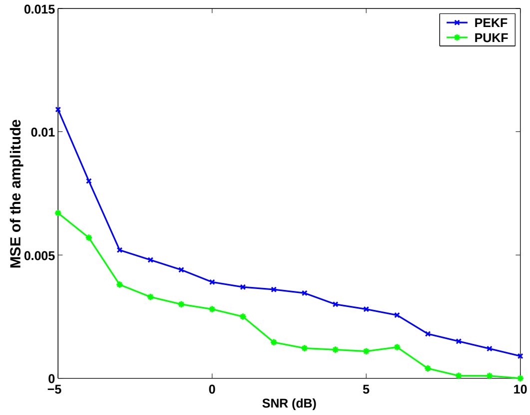

Fig. 3. Amplitude estimation

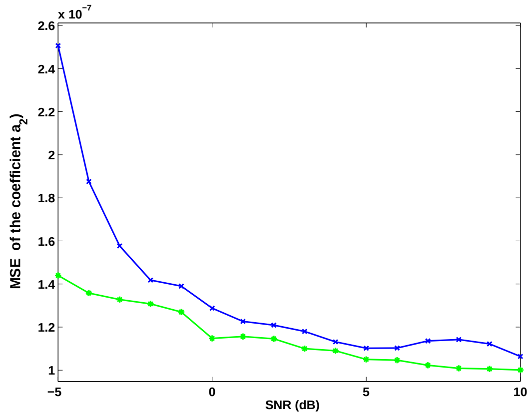

Fig. 4. Estimation of the a_2 parameter

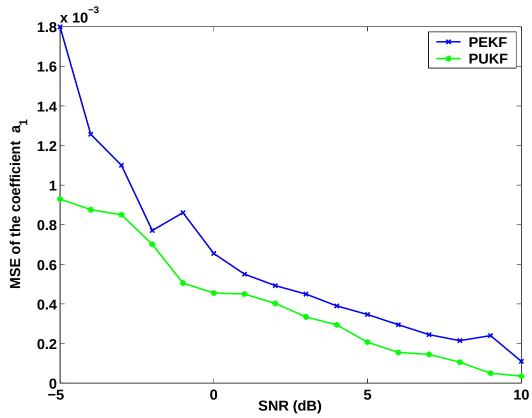

Fig. 5. Estimation of the a_1 parameter

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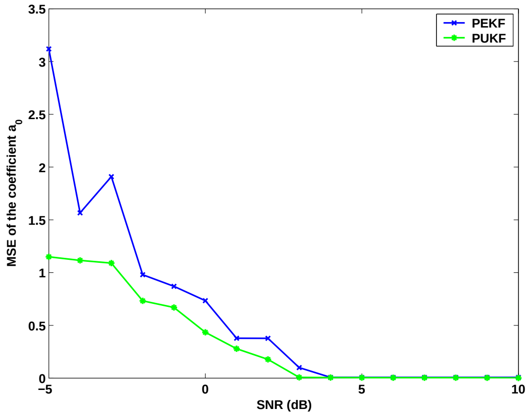

Fig. 6. Estimation of the a_0 parameter

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